Discovering **N**atural-Qualifier Space ($_{\mathbf{N}}\underline{\mathbf{Q}}$) via **N**-Cum Space ($\underline{\mathbf{C}}_{\mathbf{N}}$)

(*using differential and cumulation 'qualo-operators'*)

by

Joy-to-You

About this Brief: This 'brief' (article) is meant to answer "early-questions" (about the F.E.D.***** model) often asked by readers (including by this author). In answering those questions, both reader and author are led to "co-discover" F.E.D.'s **N**atural Qualifiers ($_{\mathbf{N}}\underline{\mathbf{Q}}$) by pathway perhaps different from the one originally discovered by Dr. Seldon. The math symbols used herein are used only for precision -- it is the questions, text, and figures which guide and explain the "co-discovery".

"Early-Questions"

When this student first began studying F.E.D.'s $_{\mathbf{N}}\underline{\mathbf{Q}}$ qualifiers and their additions and multiplications, he soon asked himself the typical *early-questions*, as you also may have asked -- questions such as:

1) Why is $_{\mathbf{N}}\underline{\mathbf{Q}} := \{\underline{\mathbf{q}}_1, \underline{\mathbf{q}}_2, \underline{\mathbf{q}}_3, \ldots\}$ in sequential correspondence with $\mathbf{N} := \{1, 2, 3, \ldots\}$?

2) Why does $\underline{\mathbf{q}}_n + \underline{\mathbf{q}}_n = \underline{\mathbf{q}}_n$?

3) Why can't the sum $\underline{\mathbf{q}}_k + \underline{\mathbf{q}}_n$ be in $_{\mathbf{N}}\underline{\mathbf{Q}}$, when $\mathbf{k} \neq \mathbf{n}$?

4) Why is the product, $\underline{\mathbf{q}}_k \times \underline{\mathbf{q}}_n = \underline{\mathbf{q}}_n + \underline{\mathbf{q}}_{n+k}$, defined as it is, necessarily with a $\underline{\mathbf{q}}_{n+k}$ term?

5) Why is the product non-commutative?

6) Why does $(\underline{\mathbf{q}}_1)^n = \underline{\mathbf{q}}_1 + \underline{\mathbf{q}}_2 + \ldots + \underline{\mathbf{q}}_n$? Was this intentional or just an elegant result?

7) "How do we know that each succeeding $\underline{\mathbf{q}}_{k+1}$ is qualitatively more definite than the previous $\underline{\mathbf{q}}_k$ when we write $\underline{\mathbf{q}}_1 \dashv\!\!\!+ \underline{\mathbf{q}}_2 \dashv\!\!\!+ \ldots \dashv\!\!\!+ \underline{\mathbf{q}}_n \dashv\!\!\!+ \ldots$?

We asked: "Why?" "Why? "Why?" just like a little kid.

And, our "adult-parent" kept answering, "Because it *works* this way."

"Because after much research, F.E.D. was led to define it in these ways."

"They have their reasons!" …

"**Because!**"

Our little child quit asking just long enough for his "parent" (you/us) to study and discover enough – enough to begin to have some answers for our "child".

In writing about F.E.D.'s model [pages **7-8**, **F.*E*.*D*.** ***Brief #3***], this author made observations (which have stayed with him) in which "$\underline{\mathbf{q}}_1$" is likened to a "quality **e**", where "quantity **e**" is the natural exponential base in "**R**eal" ("pure-quantifier") space:

a) First, $(\underline{\mathbf{q}}_1)^{n+m} = (\underline{\mathbf{q}}_1)^n \times (\underline{\mathbf{q}}_1)^m$ is an isomorphic map from **N** into **{ Cumula }**, which is analogous to "$\mathbf{exp(n)} = \mathbf{e}^n$" in the "**R**eals": $\mathbf{e}^{n+m} = \mathbf{e}^n \times \mathbf{e}^m$, and;

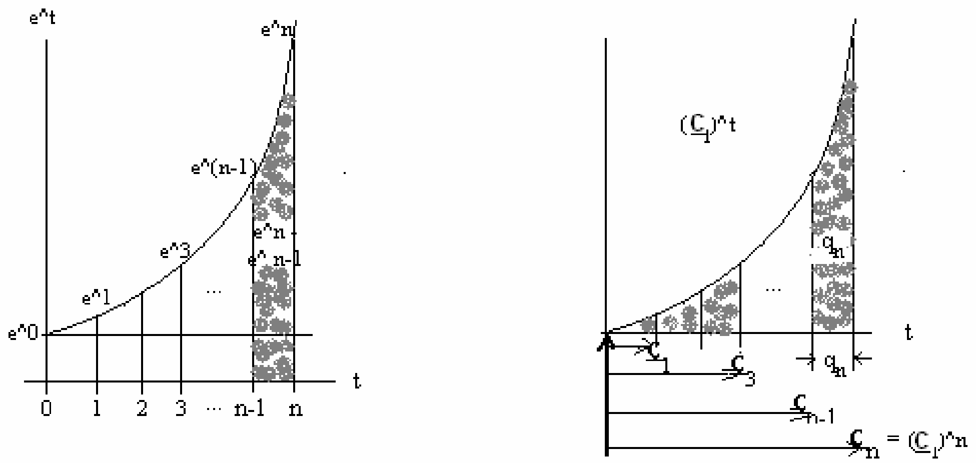
b) Second, $(\underline{\mathbf{q}}_1)^{n+m} = \underline{\mathbf{q}}_1 + \underline{\mathbf{q}}_2 + \ldots + \underline{\mathbf{q}}_2$, represents a 'Cum' or '''Sum''' function … analogous to integrating the "purely-quantitative" "**exp(t)**" over the interval **[0, n]** ("epochs" **0** to **n**), where $\int \mathbf{exp(t)dt}$ [from $\mathbf{t} = \mathbf{0}$ to **n**] "sums up" (is the "cumulative result" of) all historical (exponential) growth during those "epochs"!

c) For $\mathbf{t}$ in $\mathbf{[0, n]}$: in *Quantitative space*: $\mathbf{e^n \;=\; e^0 \;+\; \int e^t dt}$; and

in ***Qualitative** space*: $\mathbf{(\underline{q}_1)^n \;=\; q_0 \;+\; \underline{\Sigma q}_t}$.

So, not only is the "$\underline{\mathbf{q}}_1$" «***arché***»/"base" similar to the "$\mathbf{e}$" exponential base, but on the "epoch interval" $\mathbf{[0, n]}$, "$\mathbf{(\underline{q}_1)^n}$ accumulates or sums-up" all "$\underline{\mathbf{q}}_t$" qualifiers as the integral $\mathbf{\int e^t dt}$ "sums-up" all "$\mathbf{e^t dt}$" quantities! In essence, the $\mathbf{(\underline{q}_1)^n}$ function serves as both **a)** an exponential map from $\mathbf{\{n\}}$ into $\mathbf{\{\underline{q}_1{}^n\}}$, and **b)** an accumulator of qualifiers, $\underline{\mathbf{\Sigma q}}_t$ – all in one "qualo-function"! Figure **1** illustrates the "$\mathbf{e^n}$ vs. $\mathbf{(\underline{q}_1)^n}$" analogy, with each $\underline{\mathbf{q}}_t$ (as $\mathbf{t}$ increases) being regarded as "qualitatively more definite" or "more refined or polished", shall we say, than is $\underline{\mathbf{q}}_{t-1}$.

Figure **1**: Analogy between Quantitative area under $\mathbf{e^t}$ vs. Qualitative elements: $\mathbf{(\underline{C}_1)^n}$ and $\underline{\mathbf{q}}_n$.



'**Idea Space**' as '**Cumulation Space**'

Our child's questioning has now turned into a mathematician's or philosopher's design question*: "If we were to design a space of 'idea-numbers', which might serve as 'container' sets of "logical elements" or sets constituting an "idea-ontology," what properties would we require of such sets or set-numbers?"*

Since we have studied the cumulation property embodied in $\mathbf{(\underline{q}_1)^n}$ and since hindsight is always perfect, we might require that those set-numbers correspond in a direct way with the "**N**atural" numbers, and that each successive set be contained in all succeeding sets:

$\underline{\mathbf{C}}_n$ within $\underline{\mathbf{C}}_{n+1}$, or $\underline{\mathbf{C}}_n \subset \underline{\mathbf{C}}_{n+1}$ [for all $\mathbf{n}$ in $\mathbf{N}$].

This "contained-in" relationship reflects our intuitive wish that the "quality-set/-number" $\underline{\mathbf{C}}_n$ grow as $\mathbf{n}$ grows, reflecting greater cumulative "quality" or "knowledge" of some sort.

Next, we might ask that the first **N**atural "**1**" be mapped into the first 'acCumulation' set, $\underline{\mathbf{C}}_1$, and that this set might be used to generate subsequent 'acCumulation sets', just as "**1**" generates any $\mathbf{n}$ under repeated addition: $\mathbf{1 + 1 + \ldots + 1}$ ($\mathbf{n}$ times) $=$ $\mathbf{n}$ [for all $\mathbf{n}$ in $\mathbf{N}$]. [*Note*: We are "discovering cumulation" first.]

But since $\underline{\mathbf{C}}_k \subset \underline{\mathbf{C}}_n$ (for $\mathbf{k < n}$, where $\mathbf{k}$, $\mathbf{n}$ are in $\mathbf{N}$), then $\underline{\mathbf{C}}_k$ **union** $\underline{\mathbf{C}}_n$ $=$ $\underline{\mathbf{C}}_n$, this would imply a simple set-type of addition: $\underline{\mathbf{C}}_k + \underline{\mathbf{C}}_n \;=\; \underline{\mathbf{C}}_n$ when $\mathbf{k < n}$, and consequently, an idempotent addition:

$\underline{\mathbf{C}}_n + \underline{\mathbf{C}}_n \;=\; \underline{\mathbf{C}}_n$, when $\mathbf{k = n}$ (since $\underline{\mathbf{C}}_n$ **union** $\underline{\mathbf{C}}_n$ $=$ $\underline{\mathbf{C}}_n$).

Because set-number *addition* is "non-generative", we need a "set-number *multiplication*" under which set-number $\underline{\mathbf{C}}_1$ would generate set-number $\underline{\mathbf{C}}_n$. Corresponding to how $< \mathbf{1} + >$ ($\mathbf{n}$ times) generates $\mathbf{n}$ (for all $\mathbf{n}$ in $\mathbf{N}$), so $< \underline{\mathbf{C}}_1 \times >$ ($\mathbf{n}$ times) would generate $\underline{\mathbf{C}}_1 \times \underline{\mathbf{C}}_1 \times \ldots \times \underline{\mathbf{C}}_1 \;=\; (\underline{\mathbf{C}}_1)^n \;:=\; \underline{\mathbf{C}}_n$ (for all $\mathbf{n}$ in $\mathbf{N}$). This formulation also reflects our normal use of language in which we often (almost unconsciously) contend that our cumulative "knowledge grows exponentially".

These loosely-stated requirements would imply an 'exponential-$\underline{\mathbf{C}}_1$' isomorphic map $\mathbf{exQ(\_)}$ from $\mathbf{N}$ to some "acCumulation space" such that '$\mathbf{exQ(1)}$' $= \underline{\mathbf{C}}_1$, and $\mathbf{exQ(n)} = (\underline{\mathbf{C}}_1)^n \;:=\; \underline{\mathbf{C}}_n$. Thus, for all $\mathbf{n}$, $\mathbf{m}$ in $\mathbf{N}$, $(\underline{\mathbf{C}}_1)^{n+m} \;=\; (\underline{\mathbf{C}}_1)^n \times (\underline{\mathbf{C}}_1)^m$, i.e., $\mathbf{exQ(n)} \;:=\; (\underline{\mathbf{C}}_1)^n \;:=\; \underline{\mathbf{C}}_n$ and $\mathbf{exQ(n+m)} \;=\; \mathbf{exQ(n)} \times \mathbf{exQ(m)}$:

$$\mathbf{exQ(\;)}:\; \mathbf{N} \rightarrow \underline{\mathbf{C}}_{\mathbf{N}} \;:=\; \{\, \mathbf{exQ(n)} \;:=\; \underline{\mathbf{C}}_n \!:\; \underline{\mathbf{C}}_n = (\underline{\mathbf{C}}_1)^n, \text{ for all } \mathbf{n} \text{ in } \mathbf{N} \,\} \;:=\; \text{“}\underline{\mathbf{N}\text{-Cumulation space}}\text{”}.$$

Thus, our Cumulation or "**N-Cum**" space, $\underline{\mathbf{C}}_{\mathbf{N}}$, would have the following properties:

1) $< \underline{\mathbf{C}}_{\mathbf{N}}, \times, + >$ is isomorphic to $< \mathbf{N}, +, \mathbf{max(\;)} >$, where $\underline{\mathbf{C}}_k \times \underline{\mathbf{C}}_n \;:=\; \underline{\mathbf{C}}_{k+n}$, and where

$\underline{\mathbf{C}}_k + \underline{\mathbf{C}}_n \;:=\; \underline{\mathbf{C}}_{\max\{k,n\}}$ s.t. $\mathbf{max\{k, n\}} = \mathbf{n}$ if $\mathbf{n > k}$; $\mathbf{k}$ if $\mathbf{k > n}$, and s.t. each "idea-number $\underline{\mathbf{C}}_n$" behaves as a set, representing a "container set-number" for some "cumulated set of ideas".

2) $\underline{\mathbf{C}}_n + \underline{\mathbf{C}}_n \;=\; \underline{\mathbf{C}}_n$ for each $\mathbf{n}$ in $\mathbf{N}$.

3) $\underline{\mathbf{C}}_k \mathrel{-\!\!\!+} \underline{\mathbf{C}}_n \Leftrightarrow \underline{\mathbf{C}}_k \subset \underline{\mathbf{C}}_n \Leftrightarrow \mathbf{k < n}$, with the new symbol, '$\mathrel{-\!\!\!+}$', signing 'lower than' in 'total qualitative knowledge' or 'qualitative definiteness' [with '$\Leftrightarrow$' signing bi-directional implication].

4) $< \underline{\mathbf{C}}_{\mathbf{N}}$, '$\mathrel{-\!\!\!+}$' $>$ is a total order, i.e., $\underline{\mathbf{C}}_1 \mathrel{-\!\!\!+} \underline{\mathbf{C}}_2 \mathrel{-\!\!\!+} \underline{\mathbf{C}}_3 \mathrel{-\!\!\!+} \ldots$.

Note: We recognize that increasing 'total qualitative knowledge' implies a greater capacity to refine or define whatever idea-set or "ontology" is emerging. Thus, this increased "defining" results in increased 'qualitative definiteness', as $\mathbf{n}$ increases. E.g., early man looked within the jungle's (or sky's) canopy and perceived a vaguely-understood "flat earth". Only by his increased knowledge has man recognized that such a "flat earth" is merely a small surface portion of a well-defined "oblate-spheroidal Earth", i.e., human knowledge has grown qualitatively more definite – at least in regard to these "scientific matters".

Defining $_{\mathbf{N}}\underline{\mathbf{Q}}$ using the 'Differential Qualo-Operator' $\underline{\partial}$

At this point, we begin to consider the "incremental qualifier accretions" from $\underline{\mathbf{C}}_1$ to $\underline{\mathbf{C}}_2$, from $\underline{\mathbf{C}}_2$ to $\underline{\mathbf{C}}_3$, or generally, from $\underline{\mathbf{C}}_{n-1}$ to $\underline{\mathbf{C}}_n$. We define these "qualifier increments" as '$\underline{\mathbf{C}}_n \sim \underline{\mathbf{C}}_{n-1}$', where the tilde '$\sim$' denotes a "subtractive difference" as used in set notation: '$\underline{\mathbf{C}}_n \sim \underline{\mathbf{C}}_{n-1}$' or '$\underline{\mathbf{C}}_n \setminus \underline{\mathbf{C}}_{n-1}$', to denote "all elements in set $\underline{\mathbf{C}}_n$ but not in set $\underline{\mathbf{C}}_{n-1}$".

Closely related to this "incremental" or "difference set" is the notion of a "difference operator", or more precisely, a "differential operator", that operates on **N-Cum** space "Cums". This writer recalls reading (perhaps somewhere in the F.E.D. literature, citing C. Musés) where the article claims that in new number spaces the "linear" [partial] differential $\partial(\;)$ and integral $\int(\;)$ operators might be as commonplace as the four binary operations: $+, \times, -, /$. So, we eagerly endow our Cum-space, $\underline{\mathbf{C}}_{\mathbf{N}}$, with '*qualo-*' versions of such operators:

$$< \underline{\mathbf{C}}_{\mathbf{N}}, \underline{\times}, +; \underline{\partial}, \underline{\int} > \;:=\; \underline{\mathbf{N}\text{-Cumulation space}} \text{ with ‘}\underline{\text{qualo-operators}}\text{’}.$$

Without hesitation, we apply our differential 'qualo-operator' $\underline{\partial}$ on any element $\underline{\mathbf{C}}_n$ of $\underline{\mathbf{C}}_{\mathbf{N}}$ to *define* $\underline{\mathbf{q}}_n$, for $\mathbf{n}$ and $\mathbf{k}$ in $\mathbf{N}$, $\mathbf{k < n}$, as:

$$\underline{\partial}(\underline{C}_n) := \underline{\partial C}_n \big|_{[\text{with respect to}]\,\underline{C}_n} = \underline{C}_n \sim \underline{C}_{n-1} := \underline{q}_n := \underline{\partial C}_n \Leftrightarrow \underline{C}_n = \underline{C}_{n-1} + \underline{\partial C}_n = \underline{C}_{n-1} + \underline{q}_n,$$

or more generally,

$$\underline{\partial C}_n \big|_{\underline{C}_k} = \underline{C}_n \sim \underline{C}_{n-k-1} := \underline{\Sigma q}_t \big|_{t \text{ in } [n-k,\, n]} \Leftrightarrow \underline{C}_n = \underline{C}_{n-k-1} + \underline{\partial C}_n \big|_{\underline{C}_k} = \underline{C}_{n-k-1} + \underline{\Sigma q}_t \big|_{t \text{ in } [n-k,\, n]}.$$

Note: Remember that '$\underline{C}_n \sim \underline{C}_{n-1} := \underline{q}_n$' denotes an "incremental qualitative difference", which says: "$\underline{C}_n$ *without* $\underline{C}_{n-1}$ defines (is) the $\underline{q}_n$ qualifier".

Thus, $\underline{\partial}$: $\underline{C}_{\mathbf{N}} \rightarrow \{\underline{q}_n = \underline{\partial C}_n\text{: } n \text{ in } \mathbf{N}\} = {}_{\mathbf{N}}\underline{Q}$! We have defined ("co-discovered"!) that ${}_{\mathbf{N}}\underline{Q}$ is the set of 'qualo-differentials' of 'Cumulation' space elements: $\underline{\partial}[\,\underline{C}_{\mathbf{N}}\,] := {}_{\mathbf{N}}\underline{Q}$. The differential operator operating on $\underline{C}_{\mathbf{N}}$ creates ${}_{\mathbf{N}}\underline{Q}$, or ${}_{\mathbf{N}}\underline{Q}$ is "$\underline{\partial}$erived from" $\underline{C}_{\mathbf{N}}$, its 'Cumulation space'. This answers early-question **1**, as we have "co-discovered" ${}_{\mathbf{N}}\underline{Q}$ in a new way!

Now, apply our linear 'qualo-operator' $\underline{\partial}$ to the idempotent 'Cum sum': $\underline{C}_n + \underline{C}_n = \underline{C}_n$, to obtain: $\underline{\partial}(\underline{C}_n + \underline{C}_n) = \underline{\partial C}_n$ '+' $\underline{\partial C}_n = \underline{\partial C}_n$, which says $\underline{q}_n$ '+' $\underline{q}_n = \underline{q}_n$ and defines the addition '+' of ${}_{\mathbf{N}}\underline{Q}$ elements, answering early-question **2**.

Next, assume the sum $\underline{q}_k + \underline{q}_n = \underline{q}_m$ is in ${}_{\mathbf{N}}\underline{Q}$ (assuming $\mathbf{k < n}$ *and* $\mathbf{m \neq n}$, for **m**, **k**, and **n** in **N**). Then $\underline{q}_m = \underline{q}_k + \underline{q}_n = \underline{\partial C}_k + \underline{\partial C}_n = \underline{\partial}(\underline{C}_k + \underline{C}_n) = \underline{\partial}(\underline{C}_n) = \underline{q}_n$, so $\underline{q}_m = \underline{q}_n$ or $\mathbf{m = n}$, contradicting our assumption that $\mathbf{m \neq n}$! Thus, by this ***reductio ad absurdum*** proof, $\underline{q}_k + \underline{q}_n$ cannot be in ${}_{\mathbf{N}}\underline{Q}$, if $\mathbf{k < n}$, answering early-question **3**.

From the relation, $\underline{C}_n = \underline{C}_{n-1} + \underline{q}_n$, we quickly discover (proven in Appendix **A1**) that, given $\underline{C}_1 = \underline{q}_1$, $\underline{C}_2 = \underline{q}_1 + \underline{q}_2, \ldots$, and that $(\underline{q}_1)^n = (\underline{C}_1)^n = \underline{C}_n = \underline{q}_1 + \ldots + \underline{q}_n$, as the result for $\underline{C}_{\mathbf{N}}$, as previously defined by F.E.D. for $(\underline{q}_1)^n$, and answering early-question **6**. These results also lead to the following conjectures on how the multiplication ($\underline{q}_k$ "×" $\underline{q}_n$) of $\underline{q}$ elements might be defined.

First, we know that this "×" (possibly different from $\underline{\times}$ = 'Cumulation-Space-×') must satisfy the $(\underline{q}_1)^n = \underline{q}_1 + \ldots + \underline{q}_n$ relation/result [for all **n** in **N**].

Next, since $\underline{q}_k := \underline{\partial C}_k$, $\underline{q}_n := \underline{\partial C}_n$, and since $\underline{C}_k \,\underline{\times}\, \underline{C}_n := \underline{C}_{n+k}$, we necessarily have $\underline{\partial}(\underline{C}_k \,\underline{\times}\, \underline{C}_n) = \underline{\partial}(\underline{C}_{n+k}) := \underline{q}_{n+k}$. Thus, we would expect/require that the "$\underline{q}_{n+k}$" term be part of the defined product, $\underline{q}_k \,\underline{\times}\, \underline{q}_n := (\underline{\partial C}_k) \,\underline{\times}\, (\underline{\partial C}_n)$, answering part of early-question **4**.

Possible Multiplications on ${}_{\mathbf{N}}\underline{Q}$ elements

So, perhaps the most-complex of the "simple product" definitions might be [for **k**, **n** in **N**]:

$$\underline{q}_k \text{ "×" } \underline{q}_n := (\underline{\partial C}_k)(\underline{\partial C}_n) := (\underline{\partial C}_k) + (\underline{\partial C}_{n+k}) + (\underline{\partial C}_n) = \underline{q}_k + \underline{q}_{n+k} + \underline{q}_n$$

[F.E.D. calls this product-definition *"the meta-genealogical evolute product rule"*.]

So, a quick, non-exhaustive list of *possible definitions for multiplication* might be:

1) $\underline{q}_k$ "×" $\underline{q}_n := \underline{q}_{n+k}$, commutative [F.E.D. name: *"meta-heterosis convolute product"*];
2) $\underline{q}_k$ "×" $\underline{q}_n := \underline{q}_k + \underline{q}_{n+k}$, non-commutative [F.E.D. name: *"meta-catalysis evolute product"*];
3) $\underline{q}_k$ "×" $\underline{q}_n := \underline{q}_{n+k} + \underline{q}_n$, non-commutative [F.E.D. name: *"**double**-«**aufheben**» evolute product"*];
4) $\underline{q}_k$ "×" $\underline{q}_n := \underline{q}_k + \underline{q}_{n+k} + \underline{q}_n$, commutative [F.E.D. name: *"meta-genealogical evolute product"*].

Definition **1** must be ruled out immediately because it implies that:

$$(\underline{q}_1)^2 \;=\; \underline{q}_1 \text{ “}\times\text{” } \underline{q}_1 \;=\; \underline{q}_{1+1} \;=\; \underline{q}_2 \;\neq\; \underline{q}_1 + \underline{q}_2 \;=\; \underline{C}_2,$$

so Definition **1** implies that $(\underline{q}_1)^2 \neq \underline{C}_2$, which denies what is required.

The other three definitions meet our $(\underline{q}_1)^n = \underline{C}_n$ criterion, as shown in Appendix **A1**.

We might also rule out Definition **4** because it is commutative, and *we have implicitly required that* “×” *be defined with emphasis on just one of the two factors, namely* $\underline{q}_k$ “×” $\underline{q}_n$:= $\underline{q}_k$ ***“of”*** $\underline{q}_n$, *where the “of” implies that the second factor* ($\underline{q}_n$) *has “more influence.”* This would suggest Definition **3**, not Definition **2**, so we might define $\underline{q}_k$ ***“of”*** $\underline{q}_n$ [for **k**, **n** in **N**] as: $\underline{q}_k$ “×” $\underline{q}_n := \underline{q}_n + \underline{q}_{k+n}$, exactly as was done in the F.E.D. model! This answers early-questions **4**-**5**.

> *Notes:* **1)** $\underline{q}_k$ “×” $\underline{q}_n := \underline{q}_n + \underline{q}_{n+k}$ $\nleftrightarrow$ $\underline{q}_n$ “×” $\underline{q}_k := \underline{q}_k + \underline{q}_{k+n}$, i.e., these two products are “qualitatively-unequal”, ($\nleftrightarrow$), because their first terms (their “Boolean” or “conservation” terms), $\underline{q}_n$ vs. $\underline{q}_k$, if $\mathbf{n \neq k}$, differ qualitatively. **2)** *Addition* of $_{\mathbf{N}}\underline{\mathbf{Q}}$ elements (terms) ***is*** commutative (analogous to “set-union” being commutative). **3)** A *non*-commutative “×” multiplication was defined for the ‘qualo-differential’ elements (i.e., for all of the $\underline{q}_n$ in $_{\mathbf{N}}\underline{\mathbf{Q}}$) even though the ‘cum-$\underline{\times}$’ of ‘Cumulation space’ ($\underline{C}_{\mathbf{N}}$) is *commutative.*

[Only early-question **7** remains to be answered: “How do we know that each succeeding $\underline{q}_{k+1}$ is qualitatively more definite than the previous $\underline{q}_k$, as implied when we write $\underline{q}_1 \longrightarrow \underline{q}_2 \longrightarrow \ldots \longrightarrow \underline{q}_n \longrightarrow \ldots$? To answer this, we appeal to the analogy between $\mathbf{e^n}$ and $(\underline{q}_1)^n$ discussed earlier. In Figure **1** we “see” that each increment of “$\mathbf{e^t dt}$” grows and is quantitatively larger than any previous one, as **t** grows. Although we cannot directly account for the “qualitative definiteness” of each $\underline{\partial}(\underline{q}_1)^n := \underline{q}_n$, we can “reason by analogy”, and by our “*Note*” on “increasing definiteness”, p. **3**. Both support our case for increasing “qualitative definiteness” of each successive qualitative-increment (qualifier), as **n** increases. Hopefully this argument answers early-question **7** *for now*, for *generic* $_{\mathbf{N}}\underline{\mathbf{Q}}$ ‘meta-numerals’, until one gets into *specific* *models*, i.e., *interpretations*/*assignments* of the $\{\underline{q}_n\}$ to *specific* ontological categories.]

Using the ‘Cumulation Operator’ $\underline{\int}$

The ‘Cumulation operator’, or ‘qualo-integral’, $\underline{\int}(\,)$, acting on $_{\mathbf{N}}\underline{\mathbf{Q}}$ elements, produces all of **N**-**Cum**ulation space’, viz., for **n** and **k** in **N**, with **k < n**:

$$\underline{\int}:\, _{\mathbf{N}}\underline{\mathbf{Q}} \rightarrow \{\underline{C}_n\!: \mathbf{n} \text{ in } \mathbf{N}\} := \underline{C}_{\mathbf{N}}, \text{ defined by } \underline{\int}(\partial\underline{C}_n)\big|_{\text{on }[0,\,n]} := \underline{\int}(\underline{q}_n)\big|_{\text{on }[0,\,n]} := \underline{C}_n,$$

or more generally,

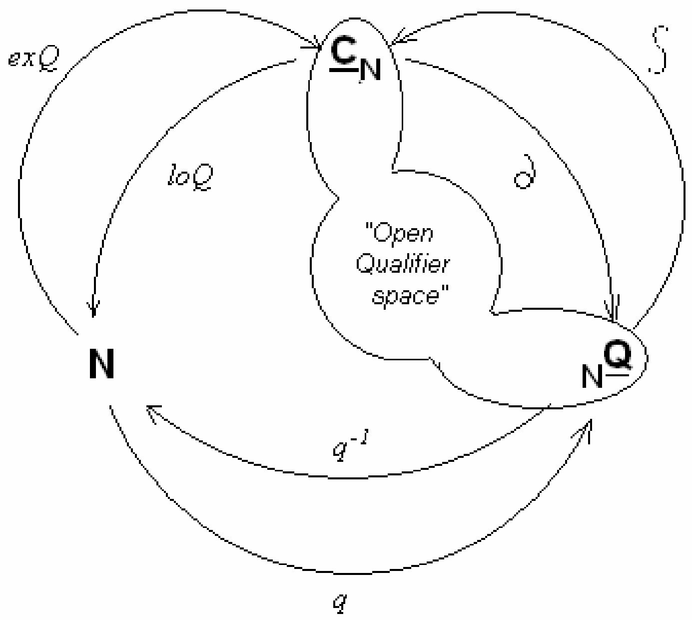
$$\underline{\int}(\partial\underline{C}_n)\big|_{\text{on }[k,\,n]} := \underline{\int}(\partial\underline{C}_n)\big|_{\text{on }[0,\,n]} \sim \underline{\int}(\partial\underline{C}_n)\big|_{\text{on }[0,\,k]} = \underline{C}_n \sim \underline{C}_k = \underline{\Sigma q}_t\big|_{t \text{ in } [k+1,\,n]}$$

So, we can define $\underline{C}_{\mathbf{N}}$ as the set that results from the ‘qualo-integration’, or ‘qualo-cumulation’, of all elements in ‘*differential space*’ $_{\mathbf{N}}\underline{\mathbf{Q}}$: $\underline{\int}(_{\mathbf{N}}\underline{\mathbf{Q}}) = \underline{C}_{\mathbf{N}}$. Thus, the cumulation operator $\underline{\int}$, operating on $_{\mathbf{N}}\underline{\mathbf{Q}}$, “resurrects” $\underline{C}_{\mathbf{N}}$!

Thus, for any **N**atural **n**, we have: $\underline{\partial}(\mathbf{exQ(n)}) = \underline{q}_n := \mathbf{q(n)}$ or, in terms of functional composition: $\underline{\partial}\mathbf{exQ(\,)} := \mathbf{q(\,)}$, the “**q**uality map” from **N** *onto* $_{\mathbf{N}}\underline{\mathbf{Q}}$. Then, the **1**:**1**-ness of these maps allows us to define inverse mappings, $\mathbf{exQ^{-1}}(\underline{C}_n) := \mathbf{loQ}(\underline{C}_n) := \mathbf{n}$ and $\mathbf{q^{-1}}(\underline{q}_n) := \mathbf{n} := \mathbf{id_N(n)}$, so: $\mathbf{loQ}(\underline{\int}\mathbf{q(n)}) = \mathbf{n} = \mathbf{q^{-1}(q(n))}$, or $\mathbf{loQ}\underline{\int} := \mathbf{q^{-1}q} := \mathbf{id_N(\,)}$, where ‘$\mathbf{id_N(\,)}$’ denotes ‘the **id**entity function’ for the elements of **N** under ‘composition of functions’.

Figure **2** summarizes functional relationships among **N**, $\underline{\mathbf{C}}_{\mathbf{N}}$, **&** $_{\mathbf{N}}\underline{\mathbf{Q}}$, via the paired inverse functions -- **q( )** and $\mathbf{q}^{-1}$**( )**, **exQ( )** and **loQ( )**, and **$\underline{\partial}$( )** and **$\underline{\int}$( )**. It also depicts "**O**pen **Q**ualifier space" as containing both $\underline{\mathbf{C}}_{\mathbf{N}}$ **&** $_{\mathbf{N}}\underline{\mathbf{Q}}$ spaces since "**OQ** space" is the space of all possible qualifier sums (including 'idempotent sums' or 'single-element sums') arising from $_{\mathbf{N}}\underline{\mathbf{Q}}$ qualifiers under addition **&** multiplication. (No, "**OQ** space" is not "like a bunny rabbit's head"! Any such resemblance simply manifests this author's limited artistic skills.)

Figure **2**: Relationships of **N**, $\underline{\mathbf{C}}_{\mathbf{N}}$, and $_{\mathbf{N}}\underline{\mathbf{Q}}$ via **q( )**, $\mathbf{q}^{-1}$**( )**, **exQ( )**, **loQ( )**, $\underline{\partial}$, and $\underline{\int}$.



Summary

We first learned F.E.D. theory as: The defined set of natural qualifiers, $_{\mathbf{N}}\underline{\mathbf{Q}}$, its properties, and its addition and multiplication operations. Then $(\underline{\mathbf{q}}_1)^{\mathbf{n}}$ was shown to be the **n**-th cumulum, and finally the set of cumula under Cum-× was shown to be isomorphic to the **N**aturals under **N** addition. Our "early-questions" about the theory led us to "answers", by "exploring-in-reverse": We first defined a "Cumulation" space $\underline{\mathbf{C}}_{\mathbf{N}}$ by an "exQ" isomorphic mapping from **N**, then, via the 'qualo-differential' operator $\underline{\partial}$, we made a "co-discovery" of $_{\mathbf{N}}\underline{\mathbf{Q}}$, its elements, its addition, and its multiplication rules!

-- *Joy-to-You* (June 2012)

***F.*E*.*D*. = Foundation *Encyclopedia Dialectica*, authors of the book *A Dialectical "Theory of Everything"* – *Meta-Genealogies of the Universe and of Its Sub-Universes*: A Graphical Manifesto, Volume 0: Foundations**. Websites providing free download of F.E.D. "primer" texts include --

www.dialectics.org and **www.adventures-in-dialectics.org**

Appendix A1 – Proofs that $(\underline{q}_1)^n = \underline{C}_n = \underline{q}_1 + \dots + \underline{q}_n$ under various defined multiplication rules

We do not know *a priori* that the **n**-th Cumulation, $\underline{C}_n$, is equal to the **n**-th Cumulum, $\underline{q}_1 + \dots + \underline{q}_n$, i.e. it must be shown that $\underline{C}_n = \underline{q}_1 + \dots + \underline{q}_n$. By definitions, $\underline{C}_n := \underline{C}_{n-1} + \partial\underline{C}_n := \underline{C}_{n-1} + \underline{q}_n$, so if we show that $\underline{C}_k = \underline{q}_1 + \dots + \underline{q}_k$, we have, by finite induction, that $\underline{C}_{k+1} := \underline{C}_k + \underline{q}_{k+1} = \underline{q}_1 + \dots + \underline{q}_k + \underline{q}_{k+1}$, i.e., that the **k+1**st-Cumulation is in fact the **k+1**st-Cumulum, given the truth of "base clause" $\underline{C}_1 = \underline{q}_1$.

Our short, non-exhaustive list of *possible definitions for* $_{\mathbf{N}}\underline{\mathbf{Q}}$ *multiplication* is:

1. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_{n+k}$;
2. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_k + \underline{q}_{n+k}$;
3. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_{n+k} + \underline{q}_n$;
4. $\underline{q}_k$ "$\times$" $\underline{q}_n := \underline{q}_k + \underline{q}_{n+k} + \underline{q}_n$.

Multiplication Definition **1** was ruled out immediately because it implied that $(\underline{q}_1)^2 \neq \underline{C}_2$, which denies what is required. We shall now show that Definitions **2** through **4** all lead to $(\underline{q}_1)^n = \underline{C}_n$ for any **n** in $\mathbf{N}$. In each case, the "base clause" $\underline{C}_1 := \underline{q}_1$ is true by definition.

Using Definition **2**, we have that $(\underline{q}_1)^2 = \underline{q}_1$ "$\times$" $\underline{q}_1 = \underline{C}_1$ "$\times$" $\underline{q}_1 = \underline{q}_1 + \underline{q}_{1+1} = \underline{q}_1 + \underline{q}_2 := \underline{C}_2$.

Assume that this generalizes to: $(\underline{q}_1)^k = \underline{C}_{k-1} \times \underline{q}_1$, for **k** in $\mathbf{N}$.

Assume that $(\underline{q}_1)^n = \underline{C}_n$ is true for $\mathbf{n} = \mathbf{k}$, i.e., that $(\underline{q}_1)^k = \underline{q}_1 + \underline{q}_2 + \dots + \underline{q}_k := \underline{C}_k$.

Then prove the "recursion clause", that $(\underline{q}_1)^{k+1} = \underline{C}_k \times \underline{q}_1 = (\underline{q}_1 + \underline{q}_2 + \dots + \underline{q}_k) \times \underline{q}_1 = \underline{C}_{k+1}$.

So, $(\underline{q}_1)^{k+1} = (\underline{q}_1 \times \underline{q}_1) + (\underline{q}_2 \times \underline{q}_1) + \dots + (\underline{q}_k \times \underline{q}_1) = (\underline{q}_1 + \underline{q}_2) + (\underline{q}_2 + \underline{q}_3) + \dots + (\underline{q}_k + \underline{q}_{k+1})$

$= (\underline{q}_1 + \underline{q}_1 + \dots + \underline{q}_1) + \underline{q}_2 + \underline{q}_3 + \dots + \underline{q}_k + \underline{q}_{k+1} = \underline{q}_1 + \underline{q}_2 + \underline{q}_3 + \dots + \underline{q}_k + \underline{q}_{k+1} = \underline{C}_{k+1}$.

Thus, by finite induction, the equation $(\underline{q}_1)^n = \underline{C}_n$ is true for all **n** in $\mathbf{N}$ for Multiplication Definition **2**.

Similarly for Definition **3**. Given $(\underline{q}_1)^n = \underline{C}_n$ for $\mathbf{n} = \mathbf{k}$, i.e. $(\underline{q}_1)^k = \underline{C}_{k-1} \times \underline{q}_1 = \underline{q}_1 + \underline{q}_2 + \dots + \underline{q}_k := \underline{C}_k$,

prove that $(\underline{q}_1)^{k+1} = \underline{C}_k \times \underline{q}_1 = (\underline{q}_1 + \underline{q}_2 + \dots + \underline{q}_{k-1} + \underline{q}_k) \times \underline{q}_1 = \underline{C}_{k+1}$.

So, $(\underline{q}_1)^{k+1} = (\underline{q}_1 \times \underline{q}_1) + (\underline{q}_2 \times \underline{q}_1) + \dots + (\underline{q}_k \times \underline{q}_1) = (\underline{q}_1 + \underline{q}_2) + (\underline{q}_1 + \underline{q}_3) + \dots + (\underline{q}_1 + \underline{q}_k) + (\underline{q}_1 + \underline{q}_{k+1})$

$= (\underline{q}_1 + \underline{q}_1 + \dots + \underline{q}_1) + \underline{q}_2 + \underline{q}_3 + \dots + \underline{q}_k + \underline{q}_{k+1} = \underline{q}_1 + \underline{q}_2 + \underline{q}_3 + \dots + \underline{q}_k + \underline{q}_{k+1} = \underline{C}_{k+1}$.

Thus, by finite induction, the equation $(\underline{q}_1)^n = \underline{C}_n$ is true for all **n** in $\mathbf{N}$ for Multiplication Definition **3**.

Finally, for Definition **4**, Given $(\underline{q}_1)^n = \underline{C}_n$ for $\mathbf{n} = \mathbf{k}$, i.e. $(\underline{q}_1)^k = \underline{C}_{k-1} \times \underline{q}_1 = \underline{q}_1 + \underline{q}_2 + \dots + \underline{q}_k := \underline{C}_k$,

prove that $(\underline{q}_1)^{k+1} = \underline{C}_k \times \underline{q}_1 = (\underline{q}_1 + \underline{q}_2 + \dots + \underline{q}_k) \times \underline{q}_1 = \underline{C}_k$.

So, $(\underline{q}_1)^{k+1} = (\underline{q}_1 \times \underline{q}_1) + (\underline{q}_2 \times \underline{q}_1) + \dots + (\underline{q}_k \times \underline{q}_1)$

$= (\underline{q}_1 + \underline{q}_2 + \underline{q}_1) + (\underline{q}_2 + \underline{q}_3 + \underline{q}_1) + \dots + (\underline{q}_k + \underline{q}_{k+1} + \underline{q}_1)$

$= (\underline{q}_1 + \underline{q}_1 + \dots + \underline{q}_1) + \underline{q}_1 + \underline{q}_2 + \underline{q}_3 + \dots + \underline{q}_k + \underline{q}_{k+1} = \underline{C}_{k+1}$.

Thus, by finite induction, the equation $(\underline{q}_1)^n = \underline{C}_n$ is true for all **n** in $\mathbf{N}$ for Multiplication Definition **4**.